Hong Kong Mathematics Olympiad (2024/25)
Finals (Individual – Event 1) (Modified by EDB)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		School ID	**FE-**	
			+ Bonus score				Time		
								Min.	Sec.
			Total score						

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. If we choose a number randomly from the factors of $13!$, the probability that it is an odd number is $\frac{1}{A}$. Find the value of A.

如果我們從 $13!$ 的因數中隨機選擇一個數，這個數是奇數的概率是 $\frac{1}{A}$，求 A 的值。

$A =$

2. If a and b are two different positive integers such that $ab = a + b + A + 1$, find the value of $B = a + b$.
如果 a 和 b 是兩個不同的正整數，使得 $ab = a + b + A + 1$，求 $B = a + b$ 的值。

$B =$

3. Let C be the number of positive integers less than or equal to $31B$ that are divisible by neither 3 nor 5. Find C.
設 C 為小於或等於 $31B$ 且既不能被 3 也不能被 5 整除的正整數的數量，求 C。

$C =$

4. Let $D = \frac{C}{53}$. Solve for the integer x such that $\log_D x + \log_D (x - 20) = 3$.

設 $D = \frac{C}{53}$。求整數解 x 使 $\log_D x + \log_D (x - 20) = 3$。

$D =$

Hong Kong Mathematics Olympiad (2024/25)
Finals (Individual – Event 2) (Modified by EDB)

FOR OFFICIAL USE

Score for accuracy		$\times$	Mult. factor for speed		$=$		School ID	**FE-**
			$+$ Bonus score				Time	Min. / Sec.
			Total score					

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. Let A be the set of all positive integers n such that n^2+2n is a perfect square and α be the number of elements of A. Find α.
設 A 為所有正整數 n 的集合，且 n^2+2n 為完全平方數及 α 為 A 中元素個數。求 α。

$\alpha =$

2. Let G be the set of positive integers which are smaller than 1000, where $\alpha+3$, $\alpha+5$ and $\alpha+7$ are not factors. Let b be the number of elements in G and $\beta=\left[\frac{b}{100}\right]$, where $[x]$ denotes the largest integer not exceeding x. Find β.
設 G 為小於 1000 的正整數集合，其中 $\alpha+3$ 、$\alpha+5$ 和 $\alpha+7$ 不是因數。設 b 為 G 的元素個數及 $\beta=\left[\frac{b}{100}\right]$，其中 $[x]$ 為不超過 x 的最大整數。求 β。

$\beta =$

3. Given that x, y and z satisfy

$$x-y+z=1$$
$$\frac{x}{2\beta}-\frac{y}{\beta}+\frac{z}{2}=1$$
$$\frac{x}{27}-\frac{y}{9}+\frac{z}{3}=1$$

Let $\gamma=\left[\frac{xyz}{100}\right]$, where $[x]$ denotes the largest integer not exceeding x. Find γ.
已知 x 、y 和 z 滿足

$$x-y+z=1$$
$$\frac{x}{2\beta}-\frac{y}{\beta}+\frac{z}{2}=1$$
$$\frac{x}{27}-\frac{y}{9}+\frac{z}{3}=1$$

設 $\gamma=\left[\frac{xyz}{100}\right]$，其中 $[x]$ 為不超過 x 的最大整數。 求 γ。

$\gamma =$

4. Let ΔABC be a triangle with sides $a = BC$, $b = AC$ and $c = AB$. The in-circle of ΔABC is tangent to BC, AC and AB at D, E and F respectively. If $BD = \gamma$, $DC = \gamma + 1$ and $AE = \gamma + 2$, find the perimeter of ΔABC.

設 ΔABC 為一個三角形，其邊為 $a = BC$、$b = AC$、$c = AB$。 ΔABC 的內切圓分別與 BC、AC 及 AB 相切於點 D、E 及 F。 若 $BD = \gamma$ 、 $DC = \gamma + 1$ 且 $AE = \gamma + 2$ ，求 ΔABC 的周長。

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		School ID	**FE-**
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			Total score					

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. Given that $x^2+y^2-27x+54y=2025$ and $A_1=x+y$, find the value of A_1.

已知 $x^2+y^2-27x+54y=2025$ 及 $A_1=x+y$，求 A_1 的值。

$A_1 =$

2. Given that $\sin\theta(\sin\theta+\cos\theta)=\dfrac{\sqrt{A_1}-1}{\sqrt{A_1}+2}$ and $A_2=\tan\theta$, where $0°\le\theta\le 90°$. Find the value of A_2.

已知 $\sin\theta(\sin\theta+\cos\theta)=\dfrac{\sqrt{A_1}-1}{\sqrt{A_1}+2}$ 及 $A_2=\tan\theta$，其中 $0°\le\theta\le 90°$，求 A_2 的值。

$A_2 =$

3. Given that x and y are positive integers which satisfy $\dfrac{20}{x}-\dfrac{2}{y}=\dfrac{1}{A_2}$. If A_3 is the sum of all possible values of x, find the value of A_3.

已知 x 及 y 為滿足方程 $\dfrac{20}{x}-\dfrac{2}{y}=\dfrac{1}{A_2}$ 的正整數。若 A_3 是所有 x 的可能值之和，求 A_3 的值。

$A_3 =$

4. In a triangle ABC, $\angle BAC=45°$, $\angle ABC=75°$, $AC=\sqrt{A_3+1}$. If A_4 is the area of the triangle, find A_4.

在三角形 ABC 中，$\angle BAC=45°$，$\angle ABC=75°$，$AC=\sqrt{A_3+1}$。若 A_4 是該三角形的面積，求 A_4。

$A_4 =$

Hong Kong Mathematics Olympiad (2024/25)

Finals (Individual – Event 4) (Modified by EDB)

FOR OFFICIAL USE

Score for accuracy		$\times$	Mult. factor for speed		$=$		School ID	**FE-**
			$+$	Bonus score			Time	Min. / Sec.
				Total score				

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. Let a be a positive real number such that $A=a+\frac{1}{a}$. If $a^3+\frac{1}{a^3}=110$, find the value of A.

設 a 為正實數，且 $A=a+\frac{1}{a}$ 。 若 $a^3+\frac{1}{a^3}=110$，求 A 的值。

$A=$

2. Given that $B=\sqrt{1+(2020+A)^2+\frac{2025^2}{2026^2}}-\frac{1}{2026}$, find the value of B.

已知 $B=\sqrt{1+(2020+A)^2+\frac{2025^2}{2026^2}}-\frac{1}{2026}$，求 B 的值。

$B=$

3. Given that C and y are positive integers and $C<y$. If $Cy-C-y=B$, find the value of C.

已知 C 及 y 是正整數且 $C<y$。若 $Cy-C-y=B$，求 C 的值。

$C=$

4. If $f(x)=\frac{9^x}{9^x+C}$, find $f\left(\frac{1}{2025}\right)+f\left(\frac{2}{2025}\right)+f\left(\frac{3}{2025}\right)+\cdots+f\left(\frac{2024}{2025}\right)$.

設 $f(x)=\frac{9^x}{9^x+C}$，求 $f\left(\frac{1}{2025}\right)+f\left(\frac{2}{2025}\right)+f\left(\frac{3}{2025}\right)+\cdots+f\left(\frac{2024}{2025}\right)$。

$D=$